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MODELING ELECTROHYDRODYNAMIC FLOWS IN SLIGHTLY CONDUCTIVE LIQUIDS

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This article is concerned with mathematical modeling of electrohydrodynamic (EHD) flows in a slightly conductive liquid exposed to an electric field and interacting with conducting and dielectric walls of the containing vessel. We determined the volt–ampere characteristic of a parallel-plate electrode system and the structure of liquid layers in proximity to the electrodes in the case of strong electric fields. The model proposed is shown to be adequate to the actual bipolar structure at electrodes [1, 2]. The structure is as follows: the layer in contact with a live electrode is of like polarity, the next layer has opposite polarity, and the third is neutral. Such a structure is responsible for the nonuniform pressure distribution in the liquid close to the electrodes. This, in turn, may bring the initially stationary liquid into motion inside the closed space.

The relaxation time of a space charge in a slightly conducting medium is of the same order or exceeds the characteristic hydrodynamic times. Thus, a space charge can appear in the medium, making it capable of electric-field-controlled motion. There are many technological and engineering applications based on the use of the interaction between a slightly conducting medium and an external electric field. An example is the so-called EHD converters intended for direct conversion of the electric-field energy into kinetic energy or the internal energy of the working agent. Among these are EHD pumps for the transfer of slightly conducting liquids such as hydrocarbon fuel or oil. Having no moving parts, these pumps are highly reliable and durable. They can be used as metering pumps since their delivery is continuously electronically adjustable even at very low magnitudes. Such a metering pump combined with an EHD sprayer can successfully replace the conventional carburetor in a computer-controlled engine.

Another promising device is an EHD compressor for refrigerating machines [3]. It also has no moving parts, is noiseless and small in size. Its most valuable advantage is the capability of handling ecologically safe coolants free of freon.

The following two important technological applications of EHD compressors should also be noted: the coating of complex-shaped surfaces by jets of powders charged in corona discharge, and the spray treatment of plants with pesticides in a strong electric field to provide sufficiently uniform distribution of charged drops over both (windward and leeward) sides of the leaves. Electric fields can successfully be used for removing bubbles from a liquid or for intensification of the coagulation of water drops suspended in a hydrocarbon liquid. EHD flowmeters for organic liquids [4] and lubricant-quality analyzers [5] are of interest too.

The possibility of using different EHD processes in numerous useful applications, some of which are mentioned above, necessitates the development of appropriate methods of mathematical modeling of flows in slightly conducting liquids interacting with external electric fields. The development of these methods involves two principal problems.

One of them arises from the fact that the media used as the working agent in an EHD-device is a technical-grade liquid dielectric. It is usually not purified and, therefore, is an electrically neutral liquid-carrier with a small indefinite amount of different electrolytes dissociated into positive and negative ions therein. The ion constitution of the working agent and, thus, the kinetic and transfer properties of its charged ingredients are not known exactly. Moreover, the medium is likely to vary in composition during experiments, e.g., because of absorption of water vapor from the atmosphere. These considerations call for the development

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of a model having a set of effective control variables that describe the kinetic and transfer properties of a whole group of charged particles of different sorts rather than individually for each sort. Groups of particles may also differ in the nature of the interaction with the conducting walls (electrodes).

The other problem springs from the difficulty of describing that interaction. A space charge is shown [6] to appear in the medium mainly due to the electrochemical interaction between the medium components and the metal electrode surface in strong electric fields. The process proceeds simply as follows. Every neutral molecule approaching negative (cathode) or positive (anode) metal electrodes takes an electron (at the cathode) or gives it (at the anode), thus becoming an ion of the same polarity as the electrode. The ions formed in the process are repelled from the like-charged surface and injected into the liquid. Due to diffusion, electric-field-forced drift, or convective flows in the transfer liquid, ions from the liquid get on the unlike-charged electrode where recombination takes place.

The discussions [7, 8] concerning injection of ions into liquid by electrodes in a strong electric field are based on the so-called phenomenological models, the parameters of which are chosen by comparing the actual and calculated volt–ampere characteristics of the medium in the electrolytic cell. Measurements were also carried out to determine the velocity profiles of flows in plane channels with a wire-plane electrode configuration [9, 10]. The model discussed below covers many of the effects found in EHD flows.

We will consider a four-component medium which consists of a neutral liquid-carrier (its individual molecules) containing three sorts of ions. The first of these are positive ions formed by electrochemical reactions alone of the electrodes, e.g., at the anode, followed by injection of the formed ions into the liquid. The other two sorts of ions form in the bulk of the solution by the dissociation of impurities into positive and negative ions. We assume that ions of these three sorts recombine at the metallic electrode surface.

Let n_i denote the concentration of i-sort ions ($i = 1, 2, 3$), let 1 correspond to positive ions injected into the volume by the anode surface, and 2, 3 to positive and negative ions formed in the volume by dissociation. The mobility factor b_i and diffusion rate D_i ($i = 1, 2, 3$), space dissociation rate w, and space recombination rate α characterize the kinetic and transfer properties of the ions.

The equations in the EHD approximation to describe the distribution of the ion concentration n_i, electric potential φ, electric intensity $\mathbf{E}$, velocity $\mathbf{u}$, and pressure p of an incompressible liquid are as follows [6]:

$$\frac{\partial n_1}{\partial t} + \operatorname{div}(n_1 \mathbf{U}_1) = 0; \tag{1}$$

$$\frac{\partial n_i}{\partial t} + \operatorname{div}(n_i \mathbf{U}_i) = w - \alpha n_2 n_3, \qquad i = 2,\ 3; \tag{2}$$

$$n_i \mathbf{U}_i = n_i \mathbf{u} - D_i \operatorname{grad} n_i \mp b_i n_i \operatorname{grad} \varphi, \qquad i = 1 - 3; \tag{3}$$

$$\varepsilon \Delta \varphi = -4\pi q, \quad \mathbf{E} = -\operatorname{grad} \varphi, \quad q = e(n_1 + n_2 - n_3); \tag{4}$$

$$\operatorname{div} \mathbf{u} = 0, \quad \rho\Big(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\nabla\mathbf{u}\Big) = -\operatorname{grad} p + q\mathbf{E} + \mu \Delta \mathbf{u}, \tag{5}$$

where $\mathbf{U}_i$ is the velocity of the i-sort component, q is the electric space density, ε is the dielectric constant of the liquid, e is the proton charge, ρ is the liquid density, and μ is the dynamic viscosity factor. Equations (1)–(5) assume that the injected ions are not involved in the space electrochemical process. In the absence of an electric field the liquid flow is entirely quasi-neutral, except for a thin diffusion layer at the electrodes, where charge separation may occur. The pulse equation for the medium as a whole takes into account only the Coulomb force. In the presence of a space charge, polarization forces were shown to have no significant effect on flows in the media under discussion. The dissociation rate w on the right-hand side of the continuity equations (2) can depend on the electric intensity in the case of strong electric fields [11].

In (1)–(5) it is assumed that the degree of ionization of the medium is small (dissociation has no significant effect on the concentration of neutral particles n_i in the medium). Therefore, there is no continuity equation for n_a in system (1)–(5). It should be noted that in the case of a highly nonuniform electric field, a space charge may appear in the medium as a result of dissociation of neutral molecules into ions by the field even if ions are not injected by electrodes [12, 13].

Let us formulate boundary conditions. The electrochemical process of formation and disappearance of ions simultaneously proceed on the metallic charged electrode surfaces. This process can be described by a balance relationship between the resultant flow of i-sort ions from the medium and the difference between the flows of these ions appearing and disappearing on the wall. For component flows normal to the wall, the equations take the following form:

$$n_i \mathbf{U}_i \cdot \boldsymbol{\nu} = a_{0i} + a_{1i}E - k_i n_i, \qquad i = 1\text{–}3, \tag{6}$$

where $\boldsymbol{\nu}$ is the vector normal to the electrode surface, k_i is rate constant of the ion surface recombination, a_{0i} and a_{1i} are the coefficients of the expansion of the ion flow injected into the liquid by the electrode surface in terms of powers of the electric intensity module. The coefficients for dielectric walls on the right-hand side of Eq. (6) should all be set equal to zero.

Boundary conditions placed on the hydrodynamic variables are conventional for problems of viscous incompressible liquid flows.

There are a number of characteristic time constants in EHD flow development problems:

$$\tau_e = \frac{L}{b_0 E_0}, \quad \tau_h = \frac{L}{u_0}, \quad \tau_r = \frac{\varepsilon}{4\pi\sigma_0}, \quad \tau_{ch} = (w_0\alpha_0)^{-1/2},$$

$$\tau_d = \frac{L^2}{D_0}, \quad \sigma_0 = 2en_0b_0, \quad n_0 = (w_0/\alpha_0)^{1/2}.$$

Here the subscript 0 denotes the characteristic quantities mentioned above, σ_0 is the quasi-neutral medium conductance, L is the characteristic dimension in the problem, and the n_0 is the quasi-neutral concentration determined from the condition of chemical equilibrium of the space dissociation of molecules into ions. The physical meaning of the characteristic time constants is as follows: τ_e is the electric-field-forced drift time, τ_h is the hydrodynamic time, τ_r is the space-charge relaxation time, τ_d is the diffusion time, τ_{ch} is the chemical-equilibrium time. With the help of the Langevin relation $\alpha_0 = 8\pi b_0/\varepsilon$, it can be shown that the time constants τ_r and τ_{ch} are the same. A change to dimensionless variables in the initial equations (1)–(5) and boundary conditions (6) give rise to dimensionless quantities:

$$\delta = \frac{\tau_r}{\tau_d}, \quad R_d = \frac{\tau_d}{\tau_h}, \quad R_q = \frac{\tau_e}{\tau_h}, \quad S_e = \frac{\varepsilon E_0^2}{8\pi\rho u_0^2}.$$

Let us estimate them for some slightly conducting liquid, transformer oil, for example, using the following values:

$$\begin{gathered} \rho = 876 \text{ kg/m}^3, \quad D_0 = 10^{-9} \text{ m}^2\text{/sec}, \quad \sigma_0 = 2.54 \cdot 10^{-11} \text{ S/m}, \\ n_0 = 2 \cdot 10^{15} \text{ m}^{-3}, \quad b_0 = 3.96 \cdot 10^{-8} \text{ m}^2/(\text{V} \cdot \text{sec}), \quad \varepsilon = 2.2, \\ L = 0.005 \text{ m}, \quad w_0 = 2.6 \cdot 10^{15} \text{m}^{-3} \cdot \text{sec}^{-1}, \quad \alpha_0 = 6.5 \cdot 10^{-16} \text{ m}^3 \cdot \text{sec}^{-1}. \end{gathered} \tag{7}$$

The characteristic time constants under these conditions are as follows: $\tau_r = 0.8$ sec, $\tau_d = 2.5 \cdot 10^4$ sec, $\tau_e = 0.04$ sec at electric intensity $E_0 = 3000$ kV/m, and $\tau_h = 0.05$ sec at flow speed $u_0 = 0.1$ m/sec. It is thus seen that the dimensionless quantities $\delta \ll 1$, $R_d \gg 1$, $R_q \approx 1$, $S_e \geqslant 1$.

The quantity δ determines the contribution of diffusion to the total ion transfer. Since it is too small, diffusion is of importance only in thin layers with large gradients of charged-particle concentration (in particular, near the walls). Beyond the range of large gradients, ion transfer is due to electric-field-forced drift and convective flows in the liquid. The contributions of these two processes are of the same order (because R_q is close to unity).

Of great importance in EHD problems [6] is also the dimensionless quantity $\theta = a_{00}/n_0 b_0 E_0$, where a_{00} is the characteristic value of the flow of ions emerging at the electrode surface. The quantity θ represents the ratio of this flow to the characteristic ion flow caused by the field-forced drift. If $\theta < 1$, all ions injected into the liquid by the electrode are completely removed from the surface into the volume under the action of

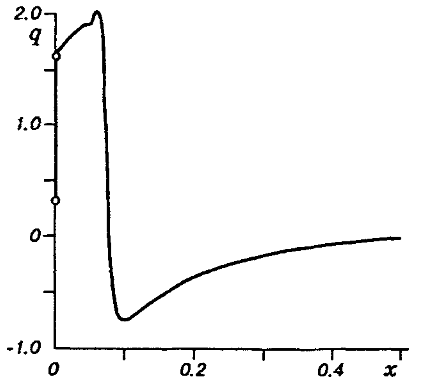

Fig. 1

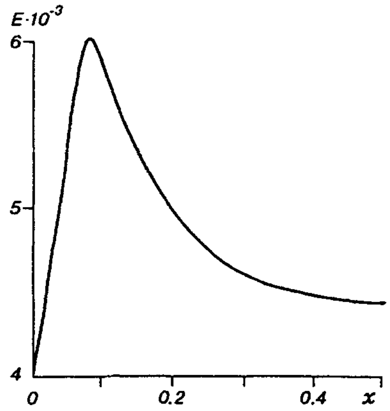

Fig. 2

the electric field. If $\theta > 1$, the field has no time for complete removal of the emerging ions from the surface, and they are accumulated in a layer near the electrode. Estimates of a_{00} and the characteristic rate of surface recombination k_{00} can be obtained from the results of experiments in [14] on the heptane flow in copper and stainless-steel tubes at low potentials: $a_{00} = 2 \cdot 10^{12}\text{m}^{-2} \cdot \text{sec}^{-1}$, $k_{00} = 10^{-3}$ m/sec.

Below, we give the results of a numerical simulation of processes developed in a constant cross-section area channel with two plane parallel electrodes filled with a low-conducting liquid after a potential difference is turned on.

Let the characteristic dimension of the electrodes placed crosswise in the channel greatly exceed the interelectrode gap L. In this case, there is no appreciable electric-field component other than E, normal to the electrode surface, and all variables depend only on time and the electrode-normal coordinate x. The liquid between the electrodes is initially assumed motionless. Hence the channel may be considered as an electrolytic cell which is commonly used in measurements of dielectric-liquid conductance. The calculations for components with subscripts 2 and 3 use the values of (7). It is assumed therewith that $b_2 = b_3 = b_0$, $b_1 < b_0$, and that ions of any sort can be absorbed by the electrodes, but the ions n_1 are the only ones to be injected (by the anode only).

Figures 1 and 2 show distributions of the space-charge density and electric intensity near the anode after all transients have completed. The potential difference is equal to $5.000kT/e$, and the injected-ion mobility $b_1 = 0.05b_0$. The abscissa is the x/L ratio (of the distance x from the anode to the interelectrode gap $L = 0.5$ cm). Plotted on the ordinate are the q/en_0 ratio and electric intensity in kT/eL units, where q is the space-charge density. The region near the anode has a complicated bipolar structure extending far outside the near-electrode diffusion layer. A thin positive-charged diffusion layer about $0.005L$ in thickness with great ion-concentration gradients is adjacent immediately to the anode. The scale of Fig. 1 allows only the extremes of q therein to be plotted (in the form of dots on the ordinate). The second layer is considerably thicker, about $0.08L$. The Coulomb forces in the positively charged layer repel the liquid from the electrodes. Since the liquid is incompressible and immovable, tensile stresses should occur in it. Note that the existence of regions with tensile stresses can lead to various cavitation effects and, hence, to a decrease in the dielectric strength of the liquid.

Next is a negatively charged layer, about $0.35L$ in thickness; the Coulomb forces acting therein tend to attract the liquid to the anode. The medium between that layer and the layer close to the cathode is quasineutral. The cathode is assumed not to inject ions, and the layer closest to it is positively charged.

With the above space-charge distribution, the electric intensity is a nonmonotone function (Fig. 2). The characteristic dimensions of the structure obtained are in good agreement with experimental data [1, 2]. It should be noted that important conditions for the formation of a bipolar structure at the anode are the

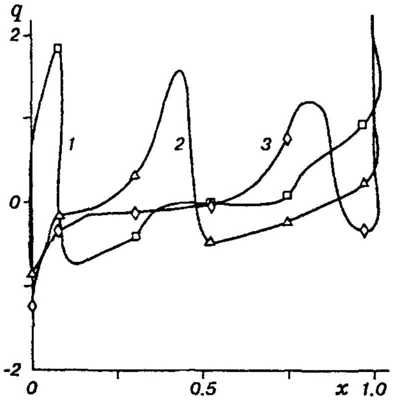

Fig. 3

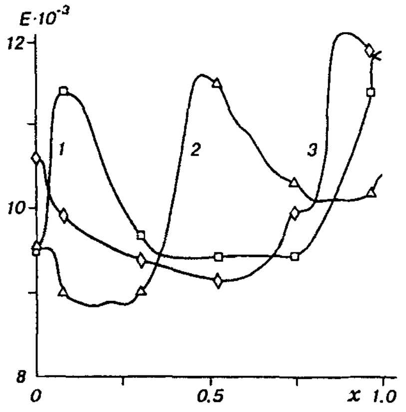

Fig. 4

presence of a nonequilibrium electrochemical process with the participation of the ions n_2, n_3 in the medium and the injection of ions n_1 (less mobile than n_2 and n_3) by the anode. If the space reaction is frozen, the bulk of the liquid quickly gets rid of ions n_2 and n_3, and the entire interelectrode gap, except for the very thin diffusion layer at the electrodes, is filled with the injected ions n_1. Also, a bipolar structure extending outside the diffusion layer does not form when $b_1 > b_0$. Thus, the processing of experimental data with the use of the proposed model provides important information about the properties of a liquid dielectric.

The nonuniform distribution of the electric intensity results in a similar (nonuniform) pressure distribution. As follows from Eq. (5), the ratio of the liquid pressure p_{00} to the pressure outside the interelectrode gap varies in proportion to the square of the dimensionless electric intensity in the case of a motionless liquid:

$$p - 1 = S_{ep}E^2, \quad S_{ep} = \frac{\varepsilon(kT)^2}{8\pi e^2 p_{00} L^2}. \tag{8}$$

Equation (8) and Fig. 2 show that the pressure has a local peak near the anode. The existence of local pressure peaks in the region of injection of charged particles is also typical of two-dimensional EHD flows in complex-shaped channels [15].

Let the electrodes now be permeable to liquid, and the medium in the channel be movable under the action of the Coulomb forces. We consider the development of the process in the interelectrode space after switching on a potential difference. The progressing convective motion in the channel substantially affects the ion concentration distribution. Figures 3 and 4 show the space-charge density and electric intensity profiles for the following three instants of time: (*1*) $0.911\tau_r$, (*2*) $1.98\tau_r$, (*3*) $2.88\tau_r$. The voltage applied equals $10^4 kT/e$, the injected-ion mobility $b_1 = 0.1b_0$. For any time moments longer than $\approx 1.5\tau_r$, the ratio of the liquid speed to the ion-drift characteristic speed b_0E is of the order of unity. Along with this, distributions created initially at the electrodes, including the diffusion layer formed at the anode, start to drift in the form of peculiar lonely waves from the anode to the cathode. In this nonstationary process, regions of steep gradients (not visible in the figures) are found not only in thin boundary layers at the electrodes, but also far away from them. The pattern obtained is not periodic in time. After the waves arise at the cathode, the steady-state distribution of the parameters is gradually established with a negative space charge at the anode and a positive space charge at the cathode. The total charge in the interelectrode gap is positive, and the Coulomb forces are balanced out by the resistance of the channel wall, which is taken into account in the hydraulic approximation. Effects similar to the described wave motions were observed in experiment [16], in which a distortion of the interference bands in an electrolytic cell was detected after voltage was turned on. Some time later, the original interference pattern was restored. Interpretation of the results of the experimental transient-current measurements in an electrolytic cell after turning the voltage on and off and

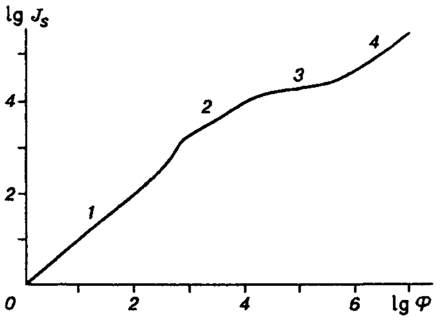

Fig. 5

after fast reversal of the electrodes is also associated with the motion of the concentration wave-fronts [17].

As was noted above, electrolytic cells are used to measure the conductivity of liquids over the linear section of the steady-state volt–ampere characteristic (VAC) at a low applied voltage. At the same time, experiments show [18] that in a wide range of voltages (up to the break-down voltage), the VAC can be divided into several characteristic sections. This provides additional important information about the medium parameters.

Figure 5 shows the cell VAC calculated within the framework of the discussed model and drawn on the bilogarithmic scale under the assumption that the medium is motionless and the ions of all sorts have the same mobility. Each point on the plot corresponds to the end of transient processes in the cell. The abscissa is the ratio of the dimensionless potential Φ to kT/e, and the ordinate is the ratio of the dimensionless current density J_s to $\sigma_0 kT/eL$.

The VAC can be divided into four sections. The conductivity of the cell σ_0 can be calculated for low potentials, using the slope of linear section *1* (equal to unity). It is governed by the ion quasineutral concentration in the liquid in the absence of field, $\sigma_0 = j_{sl}L/\varphi_w$ (j_{sl} is the measured dimensional current density within the linear section, and φ_w is the dimensional potential difference). Section *2* is transitional between the linear section *1* and section *3*. This corresponds to the saturation current at intermediate values of the potential. With very simplifying assumptions introduced into the problem, the dimensional saturation current density is as follows: $j_{sn} = 2\pi\sigma_0^2 L/\varepsilon b_0$. The effective mobility factor b_0 can easily be determined therefrom, knowing σ_0 and using the measured value of j_{sn}. A numerical solution allows one to find the inaccuracy of the given relations for b_0 and σ_0. The liquid conductivity at high applied potential differences (section 4) is governed by the ion injection. The character of the voltage–current relationship enables therewith useful information about the injection quantities to be obtained. It should be noted, however, that interpretation of the experimental data for this section seems to be impossible, since the convective ion transfer is not taken into account.

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